

EFFECTS OF HOMOGENEOUS AND HETEROGENEOUS REACTIONS ON THE DISPERSION OF A SOLUBLE MATTER IN A MHD CHANNEL FLOW

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(Received 30 August 1973)

Abstract—An analysis of the dispersion of a solute in an incompressible, viscous, electrically conducting fluid flowing between two non-conducting plates, under the action of a transverse magnetic field has been carried out on the assumption of an homogeneous irreversible first-order chemical reaction and heterogeneous reaction. Expressions for the effective Taylor diffusion coefficient have been derived in both cases. It is observed that the effective Taylor diffusion coefficient decreases due to an increase in the homogeneous reaction rate constant and also due to an increase in the heterogeneous reaction at the catalytic walls. An increase in M , the Hartmann number, leads to a decrease in the effective Taylor diffusion coefficient whereas an increase in K , the loading parameter, leads to an increase in the effective Taylor diffusion coefficient.

NOMENCLATURE

B_0 ,	magnetic induction;
C ,	concentration of the solute;
D ,	molecular diffusion coefficient;
D^* ,	Taylor diffusion coefficient;
E_y ,	electric field;
h ,	half-width of the channel;
K ,	loading parameter;
K_1 ,	first-order reaction rate constant;
L ,	characteristic length;
M ,	Hartmann number;
P ,	pressure;
Q ,	volume rate of the transport of the solute;
i ,	time, (L/\bar{u}) ;
t' ,	time;
u ,	axial velocity;
\bar{u} ,	average velocity;
u_x ,	relative velocity;
x' ,	axial co-ordinate axis;
z ,	transverse co-ordinate axis.

Greek symbols

σ ,	electrical conductivity of the fluid;
μ ,	viscosity;
θ ,	non-dimensional time;
ε ,	non-dimensional axial co-ordinate axis;
γ ,	non-dimensional number;
β ,	heterogeneous reaction rate parameter (fh).

1. INTRODUCTION

THE PROBLEM of the dispersion of a soluble matter in a viscous, incompressible fluid flowing in a circular pipe

was discussed by Taylor [1-3]. Aris [4] showed that the rate of the growth of the variance of the solute distribution is proportional to the sum of the molecular diffusion coefficient and the Taylor diffusion coefficient. Such problems being important from technological point of view, this analysis was extended to magneto-hydrodynamic channel flows of an electrically conducting fluids between non-conducting plates by Gupta and Chatterjee [5]. It was observed in [5] that the Taylor diffusion coefficient decreases with increasing the magnetic field. The effects of the conducting walls on the Taylor diffusion coefficient were studied by Soundalgekar and Gupta [6].

In all these studies, it was assumed that there is no chemical reaction between the solute and the liquid during the course of the dispersion. This is one of the possible case. However, in practice, in a number of problems of technological interest, during the process of diffusion of a solute, there does take place some kind of chemical reaction. An analysis of a finite first order homogeneous reaction in the laminar pipe flow was carried out both theoretically and experimentally by Cleland and Wilhelm [7]. The corresponding effect of a heterogeneous reaction taking place at the wall was discussed by Katz [8]. Walker [9], Soloman and Hudson [10] investigated the combined effect of the first-order heterogeneous reactions. This analysis was recently extended to the laminar channel flow by Gupta and Gupta [11].

It is the object of the present analysis to investigate the effects of the first-order homogeneous and heterogeneous reactions on the dispersion of the solute in a magnetohydrodynamic channel flow of an electrically conducting, viscous fluid between two non-conducting

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parallel plates. Now, the magnetohydrodynamic channel flow works as different device under different conditions imposed on the loading parameter. Hence, in Section 2, the expression for the velocity profiles in terms of the loading parameter is derived and following Taylor, the unsteady dispersion of a solute in the presence of a chemical reaction is studied. Expression for the effective Taylor diffusion coefficient is derived under first order homogeneous and the heterogeneous chemical reactions and numerical values for the function influencing the Taylor diffusion coefficient are calculated.

2. MATHEMATICAL ANALYSIS

Here the laminar flow of an electrically conducting, viscous, incompressible fluids is assumed between two parallel plates at $z' = \pm h$. The magnetic field is assumed to be applied parallel to the z' -axis and the x' -axis coincides with the centerline of the channel. Then the velocity profile under no-slip condition is given by

$$u = \frac{1+K}{M^2} \left(1 - \frac{\cosh Mz'}{\cosh M} \right) \quad (1)$$

where

$$\left. \begin{aligned} z = z'/h, \quad u = \frac{\mu u_1}{h^2 \left(-\frac{\partial P}{\partial x'} \right)}, \quad M = hB_0 \sqrt{(\sigma/\mu)} \\ K = \sigma E_y B_0 / (-\partial P / \partial x'). \end{aligned} \right\} \quad (2)$$

Here K is the loading parameter and M is the Hartmann number. Then the average velocity is given by

$$\bar{u} = \frac{1}{2} \int_{-1}^1 u dz = \frac{1+K}{M^2} - \frac{(1+K) \tanh M}{M^3}. \quad (3)$$

We now assume that during the course of diffusion, there is a first order irreversible chemical reaction in the liquid under isothermal conditions. Then the governing equation for the concentration C of the solute is

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x'} = D \left(\frac{\partial^2 C}{\partial x'^2} + \frac{\partial^2 C}{\partial z'^2} \right) - K_1 C \quad (4)$$

where D is the molecular diffusion coefficient, assumed constant, and K_1 is the first-order reaction rate constant. In (4), it is assumed that the solute is present in a small concentration and the last term $-K_1 C/\text{cm}^{-3}\text{s}^{-1}$ representing the volume rate of disappearance of the solute due to chemical reaction. We now assume that $\partial^2 C / \partial x'^2 \ll \partial^2 C / \partial z'^2$, and the convection is across a plane moving with the mean speed of the flow. Then

relative to this plane, the fluid velocity is given by

$$u_x = u - \bar{u} = -\frac{1+K}{M^2} \left[\frac{\tanh M}{M} - (1+K) \frac{\cosh Mz'}{\cosh M} \right]. \quad (5)$$

On introducing the following dimensionless quantities

$$\theta = t/\bar{t}, \quad \bar{t} = L/\bar{u}, \quad \varepsilon = (x - \bar{u}t)/L, \quad z = z'/h$$

in (4), we have

$$\frac{1}{\theta} \frac{\partial C}{\partial \theta} + \frac{u_x}{L} \frac{\partial C}{\partial \varepsilon} = \frac{D}{h^2} \frac{\partial^2 C}{\partial z^2} - K_1 C \quad (6)$$

where L is a typical length along the direction of the flow. Assuming Taylor's limiting condition, viz. the partial equilibrium over any cross section of the channel, we have from (6),

$$\frac{\partial^2 C}{\partial z^2} - \frac{K_1 h^2}{D} C = \frac{h^2}{DL} u_x \frac{\partial C}{\partial \varepsilon} \quad (7)$$

where $\partial C / \partial \varepsilon$ may be taken as independent of z .

Substituting for u_x from (5) in (7) and integrating twice, the solution of (7) satisfying the conditions $\partial C / \partial z = 0$ at $z = \pm 1$ is

$$C = \frac{h^2}{DL} \frac{\partial C}{\partial \varepsilon} \left[\frac{(1+K)^2}{\gamma M} \frac{\tanh M}{(M^2 - \gamma^2) \sinh \gamma} \cosh \gamma z - \frac{1+K}{M^2} \left\{ \frac{\tanh M}{M\gamma^2} + \frac{1+K}{M^2 - \gamma^2} \frac{\cosh Mz'}{\cosh M} \right\} \right] \quad (8)$$

where $\gamma = h(K_1/D)^{1/2}$.

The volumetric rate at which the solute is transported across a section of the channel of unit breadth is

$$Q = \int_{-1}^1 C u_x dz. \quad (9)$$

Substituting for C and u_x from (8) and (5) respectively in (9) and carrying out the integration, we have

$$Q = \frac{h^2}{DL} \frac{\partial C}{\partial \varepsilon} \left\{ \frac{(1+K)^3}{M^3 \gamma} \frac{\tanh M}{(M^2 - \gamma^2) \sinh \gamma} \left[\frac{2 \sinh \gamma \tanh M}{M \gamma} - \frac{1+K}{\cosh M} \left(\frac{\sinh(M+\gamma)}{M+\gamma} + \frac{\sinh(M-\gamma)}{M-\gamma} \right) \right] - \frac{2(1+K)^2 \tanh^2 M}{M^6 \gamma^2} + \frac{2(1+K)^3 \tanh^2 M}{M^6} - \frac{2(1+K)^3 \tanh^2 M}{M^6 (M^2 - \gamma^2)} + \frac{(1+K)^4}{M^4 (M^2 - \gamma^2) \cosh^2 M} \left(\frac{\sinh 2M}{2M} + 1 \right) \right\}. \quad (10)$$

On comparing (10) with Fick's law of diffusion, the solute is observed to be dispersed relative to a plane with the mean speed of the flow with effective dispersion coefficient D^* given by

$$D^* = \frac{h^2}{D} F(M, \gamma) \quad (11)$$

where

$$\begin{aligned}
 F(M, K, \gamma) = & \frac{(1+K)^3 \tanh M}{M^3 \gamma (M^2 - \gamma^2) \sinh \gamma} \\
 & \times \left[\frac{1+K}{\cosh M} \left(\frac{\sinh(M+\gamma)}{M+\gamma} + \frac{\sinh(M-\gamma)}{M-\gamma} \right) \right. \\
 & \left. - \frac{2 \sinh \gamma \tanh M}{M \gamma} \right] \\
 & + \frac{2(1+K)^2 \tanh^2 M}{M^6 \gamma^2} - \frac{2(1+K)^3 \tanh^2 M}{M^6} \\
 & + \frac{2(1+K)^3 \tanh^2 M}{M^6 (M^2 - \gamma^2)} \\
 & - \frac{(1+K)^4}{M^4 (M^2 - \gamma^2) \cosh^2 M} \left(\frac{\sinh 2M}{2M} + 1 \right)
 \end{aligned}
 \tag{12}$$

The numerical values of $F(M, K, \gamma)$ are entered in Table 1. In calculating the values of $F(M, K, \gamma)$, the values of M and γ are chosen arbitrarily whereas those of K are chosen to represent the physical nature of the magnetohydrodynamic channel. The channel configuration depends upon K as follows:

1. $K = 0$, (Short-circuited)
2. $K < 1.0$, (MHD generator)
3. $K = 1.0$, (MHD flowmeter)
4. $K > 1.0$, (MHD accelerator).

We now give the quantitative estimation of the changes that take place due to a change in the values of the parameter. Thus, in a mhd generator ($K < 1.0$), we observe that for $K = 0.2$ and γ increased from 0.2 to 0.6, there is about 92 per cent decrease in the value of $F(M, K, \gamma)$ for all M whereas for $K = 0.5$ or 0.8, there is a 91.6 per cent decrease. However, an increase in K leads to an increase in $F(M, K, \gamma)$ in a MHD generator. In a MHD accelerator, for $K = 2$, when γ is increased from 0.2 to 0.6, there is a 89.4 per cent decrease in the effective Taylor diffusion coefficient. Also it increases with increasing K . However, in all these devices, there is a fall in the effective Taylor diffusion coefficient due to an increase in M .

Diffusion with combined homogeneous and heterogeneous chemical reaction

In this case, we assume that in addition to a first-order irreversible chemical reaction in the bulk of the fluid, there is also similar one at the walls which are assumed to be catalytic. Then under Taylor's limiting conditions, equation (7) still governs the diffusion of the solute. The boundary conditions at the walls are

$$\frac{\partial C}{\partial z} \pm fC = 0 \quad \text{at } z = \pm h \tag{13}$$

which in non-dimensional form are

$$\frac{\partial C}{\partial z} \pm \beta C = 0 \quad \text{at } z = \pm 1 \tag{14}$$

Table 1. Values of $F(K, \gamma, M)$

K	γ	M		
		5	10	15
0.2	0.2	0.552167×10^{-2}	0.866643×10^{-4}	0.762252×10^{-5}
	0.6	0.442673×10^{-3}	0.728166×10^{-5}	0.653149×10^{-6}
	1.0	0.348190×10^{-4}	0.891389×10^{-6}	0.916839×10^{-7}
	1.4	-0.791929×10^{-4}	-0.911847×10^{-6}	-0.672258×10^{-7}
0.5	0.2	0.122389×10^{-1}	0.192177×10^{-3}	0.169060×10^{-4}
	0.6	0.103875×10^{-2}	0.171212×10^{-4}	0.153699×10^{-5}
	1.0	0.138996×10^{-3}	0.301999×10^{-5}	0.297912×10^{-6}
	1.4	-0.112907×10^{-3}	-0.969173×10^{-3}	-0.537702×10^{-7}
0.8	0.2	0.246985×10^{-1}	0.387870×10^{-3}	0.341232×10^{-4}
	0.6	0.221108×10^{-2}	0.363954×10^{-4}	0.326548×10^{-5}
	1.0	0.404326×10^{-3}	0.807666×10^{-5}	0.777039×10^{-6}
	1.4	-0.101777×10^{-3}	0.586331×10^{-7}	0.700823×10^{-7}
1.0	0.2	0.376002×10^{-2}	0.590484×10^{-3}	0.519485×10^{-4}
	0.6	0.346624×10^{-2}	0.569788×10^{-4}	0.510910×10^{-5}
	1.0	0.723694×10^{-3}	0.139891×10^{-4}	0.133173×10^{-5}
	1.4	-0.445921×10^{-4}	0.181675×10^{-5}	0.258466×10^{-6}
2.0	0.2	0.199277	0.312883×10^{-2}	0.275236×10^{-3}
	0.6	0.200754×10^{-1}	0.327940×10^{-3}	0.293326×10^{-4}
	1.0	0.567933×10^{-2}	0.102320×10^{-3}	0.950733×10^{-5}
	1.4	0.164885×10^{-2}	0.384932×10^{-4}	0.388042×10^{-5}
3.0	0.2	0.660984	0.103757×10^{-1}	0.912646×10^{-3}
	0.6	0.693383×10^{-1}	0.112850×10^{-2}	0.100787×10^{-3}
	1.0	0.218172×10^{-1}	0.383826×10^{-3}	0.353544×10^{-4}
	1.4	0.852175×10^{-2}	0.173395×10^{-3}	0.168061×10^{-4}

Table 2. Values of $F(K, \gamma, \beta, M)$

K	γ	β	M			
			5	10	15	
0.2	0.2	3	0.532697×10^{-2}	0.835229×10^{-4}	0.734325×10^{-5}	
		5	0.532596×10^{-2}	0.835065×10^{-4}	0.734179×10^{-5}	
		7	0.532552×10^{-2}	0.834994×10^{-4}	0.734116×10^{-5}	
		9	0.532527×10^{-2}	0.834955×10^{-4}	0.734081×10^{-5}	
	0.6	0.6	11	0.532512×10^{-2}	0.834930×10^{-4}	0.734059×10^{-5}
			3	0.411932×10^{-3}	0.669135×10^{-5}	0.597193×10^{-6}
			5	0.410691×10^{-3}	0.666752×10^{-5}	0.594934×10^{-6}
			7	0.410128×10^{-3}	0.665671×10^{-5}	0.593910×10^{-6}
		1.0	3	0.176825×10^{-4}	0.504399×10^{-6}	0.529334×10^{-7}
			5	0.161728×10^{-4}	0.470296×10^{-6}	0.495186×10^{-7}
			7	0.154405×10^{-4}	0.453768×10^{-6}	0.478636×10^{-7}
0.5	0.2	3	0.104125×10^{-1}	0.163163×10^{-3}	0.143423×10^{-4}	
		5	0.104029×10^{-1}	0.163012×10^{-3}	0.143290×10^{-4}	
		7	0.103988×10^{-1}	0.162946×10^{-3}	0.143232×10^{-4}	
	0.6	3	0.813157×10^{-3}	0.131407×10^{-4}	0.117089×10^{-5}	
		5	0.804051×10^{-3}	0.129800×10^{-4}	0.115611×10^{-5}	
		7	0.799920×10^{-3}	0.129071×10^{-4}	0.114941×10^{-5}	
	1.0	3	0.434586×10^{-4}	0.110087×10^{-5}	0.112973×10^{-6}	
		5	0.350394×10^{-4}	0.931748×10^{-5}	0.966760×10^{-7}	
		9	0.309591×10^{-4}	0.849786×10^{-6}	0.887776×10^{-7}	
	0.8	0.2	3	0.180338×10^{-1}	0.282422×10^{-3}	0.248205×10^{-4}
			5	0.179990×10^{-1}	0.281892×10^{-3}	0.247720×10^{-4}
			7	0.179840×10^{-1}	0.281634×10^{-3}	0.247510×10^{-4}
0.8	0.6	3	0.144209×10^{-2}	0.231760×10^{-4}	0.206137×10^{-5}	
		5	0.141105×10^{-2}	0.226424×10^{-4}	0.201277×10^{-5}	
		7	0.139696×10^{-2}	0.224003×10^{-4}	0.199072×10^{-5}	
	1.0	3	0.106794×10^{-3}	0.235666×10^{-5}	0.234120×10^{-6}	
		5	0.805741×10^{-4}	0.185260×10^{-5}	0.186277×10^{-6}	
		7	0.678671×10^{-4}	0.160830×10^{-5}	0.163090×10^{-6}	
	1.0	3	0.106794×10^{-3}	0.235666×10^{-5}	0.234120×10^{-6}	
		5	0.805741×10^{-4}	0.185260×10^{-5}	0.186277×10^{-6}	
		7	0.678671×10^{-4}	0.160830×10^{-5}	0.163090×10^{-6}	
	1.0	0.2	3	0.247876×10^{-1}	0.388039×10^{-3}	0.340981×10^{-4}
			5	0.247207×10^{-1}	0.386981×10^{-3}	0.340048×10^{-4}
			7	0.246757×10^{-1}	0.386271×10^{-3}	0.339646×10^{-4}
0.6		3	0.202206×10^{-2}	0.323780×10^{-4}	0.287634×10^{-4}	
		5	0.196377×10^{-2}	0.313852×10^{-4}	0.298622×10^{-4}	
		7	0.193732×10^{-2}	0.309347×10^{-4}	0.274533×10^{-4}	
1.0		3	0.182924×10^{-3}	0.376457×10^{-5}	0.366941×10^{-6}	
		5	0.135269×10^{-3}	0.286355×10^{-5}	0.281921×10^{-6}	
		7	0.112174×10^{-3}	0.242688×10^{-5}	0.240717×10^{-6}	
2.0		0.2	3	0.847487×10^{-1}	0.132412×10^{-2}	0.116278×10^{-3}
			5	0.841506×10^{-1}	0.131469×10^{-2}	0.115448×10^{-3}
			7	0.838924×10^{-1}	0.131062×10^{-2}	0.115089×10^{-3}
	0.6	3	0.776954×10^{-2}	0.122370×10^{-3}	0.108095×10^{-4}	
		5	0.727284×10^{-2}	0.114073×10^{-3}	0.100619×10^{-4}	
		7	0.704748×10^{-2}	0.110308×10^{-3}	0.972267×10^{-5}	
2.0	1.0	3	0.138286×10^{-2}	0.240358×10^{-4}	0.221791×10^{-5}	
		5	0.100425×10^{-2}	0.171371×10^{-4}	0.157555×10^{-5}	
		7	0.820751×10^{-3}	0.137938×10^{-4}	0.126423×10^{-5}	

where $\beta = fh$ is now the heterogeneous rate parameter corresponding to catalytic reaction at the walls. Then the solution of (7) satisfying (14) is given by

$$C = \frac{h^2}{DL} \left(\frac{\partial C}{\partial \varepsilon} \right) \left[\frac{\frac{(1+K)^2}{M(M^2-\gamma^2)} \tanh M + \frac{\beta(1+K)}{M^2} \left(\frac{\tanh M}{M\gamma^2} + \frac{1+K}{M^2-\gamma^2} \right)}{\gamma \sinh \gamma + \beta \cosh \gamma} \cosh \gamma z - \frac{1+K}{M^2} \left\{ \frac{\tanh M}{M\gamma^2} + \frac{(1+K) \cosh Mz}{(M^2-\gamma^2) \cosh M} \right\} \right]. \quad (15)$$

Proceeding as before, we now have the effective dispersion coefficient D^* given by

$$D^* = \frac{h^2}{D} F(M, K, \gamma, \beta)$$

where

$$F(M, K, \gamma, \beta) = \left[\frac{(1+K)^3 \tanh M}{M^3(M^2-\gamma^2)} + \frac{\beta(1+K)^2 \left(\frac{\tanh M}{M\gamma^2} + \frac{1+K}{M^2-\gamma^2} \right)}{M^4} \right] \times \left[\frac{1+K}{\cosh M} \left(\frac{\sinh(M+\gamma)}{M+\gamma} + \frac{\sinh(M-\gamma)}{M-\gamma} \right) - \frac{2 \sinh \gamma \tanh M}{M\gamma} \right] + \frac{2(1+K)^2}{M^6\gamma^2} \tanh^2 M - \frac{2(1+K)^3 \tanh^2 M}{M^6} + \frac{2(1+K)^3 \tanh^2 M}{M^6(M^2-\gamma^2)} - \frac{(1+K)^4}{M^4(M^2-\gamma^2) \cosh^2 M} \left(\frac{\sinh 2M}{2M} + 1 \right).$$

The numerical values of $F(M, K, \gamma, \beta)$ are entered in Table 2. We observe from this table that the effects of K and M are the same as described above, the effects of the heterogeneous reactions at the wall are more prominent at large values of γ , i.e. the decrease in D^* being more as β increases at large γ .

Acknowledgement—We are grateful to the Atomic Energy of India for the award of a research grant to carry out this research.

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